International Journal of Robotics
Research and Development (IJRRD)
ISSN(P): 2250-1592; ISSN(E): 2278–9421
Vol. 4, Issue 2, Apr 2014, 9-16

GRAPHICAL ANALYSIS OF PASSIVE DYNAMIC BIPED ROBOT

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ABSTRACT

It is simple for humankind to steadily walk on different terrain, but it is hard to achieve a human-like gait for bipedal walking robots due to their complex dynamics. In general, there are two approaches towards controlling a biped robot: static and dynamic walking. In this paper, we demonstrate the dynamic walking approach for controlling a biped robot. In this approach, the walker moves only under the gravitational force. The loss energy of during the walk will recover only by the gravity. The walker will have a stable gait over the course of several steps for that reason there is no need for PDBR to be stabilized in each of its steps. It can stably walk over a gentle slope. In this paper, we explain the steps of mathematical modeling which analogues to a double inverted pendulum, the impact equations for heel-strike and the stepwise analysis of walking of a passive biped. This paper shows the graphical approach to analysis the symmetric gait for the linear model of passive dynamic bipedal robot (PDBR).

KEYWORDS: Biped Robots, Passive Walking, Linearization, Switched Conditions, Compass Gait

INTRODUCTION

Biped locomotion is one of the most complex forms of legged motion. In the sense of dynamics systems, human locomotion stands out among other forms of biped locomotion mainly because of the fact that throughout a substantial part of the human walking cycle the moving body is not in the static equilibrium. Bipedal walking might be fundamentally known as a passive mechanical process, as shown for part of a stride by Mochon and MacMahon[1]. McGeer[2] studied the dynamic of passive walking, and explained a two legged robot excluding actuators and controllers can walk stably down on a ramp with a shallow slope in simulations and experiments. Gracia[3] analysed that the walking motion of the passive walker depends on the slope angle. If the slope of the ramp decreases, then walking speed also decreases. Moreover, increasing the slope angle brings about a period doubling bifurcation leading to chaotic gaits and there are only unstable gaits in high speed region. Collins, Wisse and Ruina [4] demonstrated 3D passive-dynamic walker with knees and arms. Passive-dynamic walking is functional for learning efficient level-ground walking robots, but it has some limitations explained by [5]. Many other researchers have studied about this topic [6-9].

In this paper, we study the behavior of passive dynamic walking of simple, 2D, uncontrolled, two-link model, vaguely resembling human legs, bipedal robot, which moves down on the ramp with angle α, using the graphical method. It shows walking modes similar to other models, but allows some use of graphical methods to examine its dynamics.We analyze the walking of passive dynamic stepwise by considering jump conditions immediately after the impact of swing leg with the ground and switching the velocities between swing leg and stance leg.

Mathematical Modeling

This model is the simplest model of passive dynamic robots, consisting two rigid legs pivoted at the hip, three point-masses: one at the hip and other two at the centre of mass of legs. The feet have plastic (no-slip, no-bounce) collisions with the surface of inclined ramp, except for the period of forward swinging, as geometric interference is ignored.

The mathematical modeling of biped robots is necessarily hybrid, consisting of ordinary differential equations to describe the swing phase of the walking motion, and a discrete map to model the impact when the leg touches the ground. Biped robots exhibit periodic behavior. Distinct events, such as contact with the ground, can act to trap the evolving system state within a constrained region of the state space. Therefore Limit cycles (periodic behavior) are often created in this way.

Our working model of a passive dynamic bipedal robot (PDBR) is shown in Figure 1 schematically.

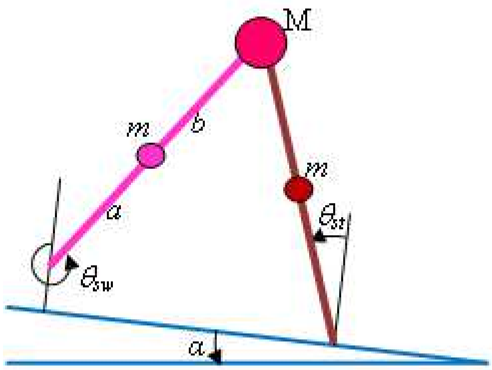

Figure 1: A Passive Dynamic Biped Robot

Table 1: Lists Physical Parameters of this Model

Symbols	Parameters
θ_{st}	Angle of stance leg with vertical upward axis
θ_{sw}	Angle of swing leg with vertical upward axis
a	The length of rod from the CM of Stance (and Swing)leg to the foot
b	The length of rod from the CM of Stance (and Swing)leg to the Hip
l	Length of rod== $a + b$
m	Mass of leg
M	Mass of Hip
α	Slope of ramp
φ	Inter legs Angle

Table 1: List of Parameters of PDBR Model

Using Euler-Lagrange equations, the dynamic equation of the robot can be derived as:

$$M(\theta)\ddot{\theta}+N(\theta,\dot{\theta})\dot{\theta}+G(\theta,\alpha)=0 \tag{1}$$

where $\theta=[\theta_{st} \quad \theta_{sw}]^T$, $M(\theta)$ is the inertia matrix, the matrix $N(\theta,\dot{\theta})$ contains terms of centrifugal and coriolis forces, $G(\theta,\alpha)$ is the gravity term as given below:

$$M(\theta)=\begin{bmatrix} ma^2+Ml^2+ml^2 & -mlb\cos(\theta_{st}-\theta_{sw}) \\ -mlb\cos(\theta_{st}-\theta_{sw}) & mb^2 \end{bmatrix};$$

$$N(\theta,\dot{\theta})=\begin{bmatrix} 0 & -mlb\sin(\theta_{st}-\theta_{sw})\dot{\theta}_{sw} \\ mlb\sin(\theta_{st}-\theta_{sw})\dot{\theta}_{st} & 0 \end{bmatrix};$$

$$G(\theta,\alpha)=\begin{bmatrix} (-ma-Ml-ml)g\sin(\theta_{st}-\alpha) \\ mgb\sin(\theta_{sw}-\alpha) \end{bmatrix}$$

Linearized State Space Model

The above non-linear model can be linearized at the equilibrium point $0_e=[0.035\ \ 0.035\ 0\ 0]^T$, the equation becomes

$$M_{0_e}\ddot{\theta}+G_{0_e}\theta=0$$

The linerized state space model can be written as

$$\dot{y}=Ay \tag{2}$$

where $y=x-0_e$, $x=\begin{bmatrix}\theta_{st} & \theta_{sw} & \dot{\theta}_{st} & \dot{\theta}_{sw}\end{bmatrix}^T$ and the elements of matrix A are:

$$A=\begin{bmatrix} 0_{2\times2} & I_{2\times2} \\ -M_{\theta_e}^{-1}G_{\theta_e} & 0_{2\times2} \end{bmatrix} \text{ where } M_{0_e}^{-1}G_{0_e}=\begin{bmatrix} \dfrac{(-ma-Ml-ml)g}{(ma^2+Ml^2)} & \dfrac{mlg}{(ma^2+Ml^2)} \\ \dfrac{l(-ma-Ml-ml)g}{b(ma^2+Ml^2)} & \dfrac{(ma^2+Ml^2+ml^2)g}{b(ma^2+Ml^2)} \end{bmatrix}.$$

Heel Strike and Impact Equations

The heel strike occurs when the swing leg touches the ramp surface. We assumed that the heel strike to be inelastic and without slipping, and the stance-leg lifts from the ramp without interaction. This impact occurs when the geometric condition, the *y*-coordinate of a foot of the swing leg will become a zero is met, that is,

$$(\theta_{st}-\alpha)+(\theta_{sw}-\alpha)=0.$$

The angular momentums are conserved at the time of impact for the new stance-leg about the foot and the hip of PDBR (see figure 2).

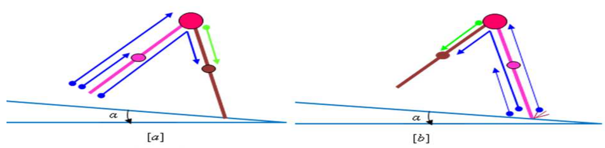

Figure 2[*a*]: Directions of Angular Momentums about the Swing Leg's Foot and the Hip before the Heel Strike [*b*] Directions of Angular Momentums about the New Stance Leg's Foot and after the Heel Strike

The conservation law of the angular momentum gives to the following compressed equation between the pre- and post-impact angular velocities after the heel strike:

$$\dot{\theta}^{+} = K(\varphi)\dot{\theta}^{-}$$

where $\theta_{sw} - \theta_{st} = \varphi$ and $K(\varphi) = \left[V^{+}(\varphi)\right]^{-1} V^{-}(\varphi)$

where

$$V^{-}(\varphi) = \begin{bmatrix} -mab + \left(2mla + Ml^2\right)\cos\varphi & -mab \\ -mab & 0 \end{bmatrix}$$

$$V^{+}(\varphi) = \begin{bmatrix} ma^2 + Ml^2 + ml^2 - mlb\cos\varphi & mb^2 - mlb\cos\varphi \\ -mlb\cos\varphi & mb^2 \end{bmatrix}$$

After the heel strike the swing leg will become the new stance and the stance leg will be the new swing leg that change can be computed using the following equation:

$$\theta^{+} = J\theta^{-} \quad \text{where } J = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Therefore the transition matrix of the state-space system can be written as:

$$y^{+} = T(\varphi)\, y^{-} \text{ where } T(\varphi) = \begin{pmatrix} J & 0 \\ 0 & K(\varphi) \end{pmatrix} \tag{3}$$

The Complete PDBR Model

The complete biped passive dynamic robot can be described as follows:

$$\begin{cases} \text{Motion equation:} & \dot{y} = Ay \\ \text{Impact equation:} & y^{+} = T(\varphi)\, y^{-}, \quad \text{when } (\theta_{st} - \alpha) + (\theta_{sw} - \alpha) = 0 \end{cases}$$

Figure 3 shows the steps of walking cycle of PDBR.

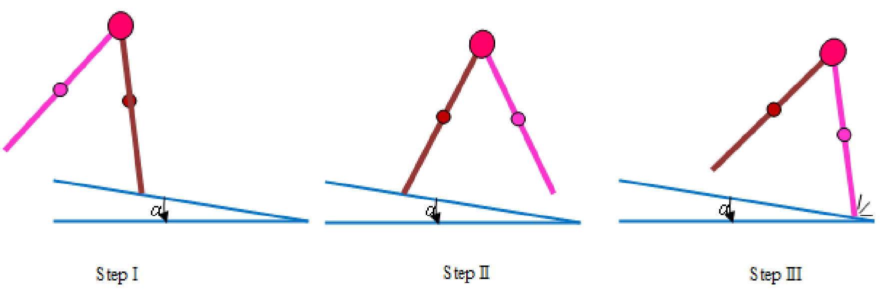

Figure 3: Walking Steps for PDBR

Analytical Procedure

The system of PDBR is hybrid of two discrete events: motion equation (2) and transition function (3). The

analytical solution of the differential equation of motion (2) is given by the explicit equation

$$y(t) = e^{At} y_0$$

where y_0 is the initial state-space position of PDBR. If τ denotes the time of heel strike when the foot of swing leg touch the ramp-surface, then the state-vector y at the time of heel strike, can be expressed as

$$y(\tau) = e^{A\tau} y_0.$$

At the time of impact, the legs will change their positions, that is, the swing leg will be the stance leg and the stance will become swing leg for the next step. This change will be done by the transition equation (3) and the state-space position of PDBR for the next step can be worked out by

$$y^+(\tau) = T(\varphi) y(\tau).$$

Considering the state-vector $y^+(\tau)$ as the initial position vector for the next step and repeating the above process with it.

RESULTS OF SIMULATION

For the simulation, the assumed the values of parameters are listed in the following Table-2.

Table 2: List of Values Parameters

Symbols	Parameters	Values
a	The length of rod from the C.M. of Swing (Stance)leg to the foot	0.5 m
b	The length of rod from the C.M. of Swing (Stance)leg to the Hip	0.5 m
l	Length of rod== $a + b$	1 m
m	Mass of leg	5 kg
M	Mass of Hip	10 kg
α	Slope of ramp	0.035 rad. (2^o)

If PDBR starts with the initial state-space position y_0 = [0.1029; –0.1248; –0.5407;–0.2918], the foot position of the swing leg of PDBR is shown the figure 4.

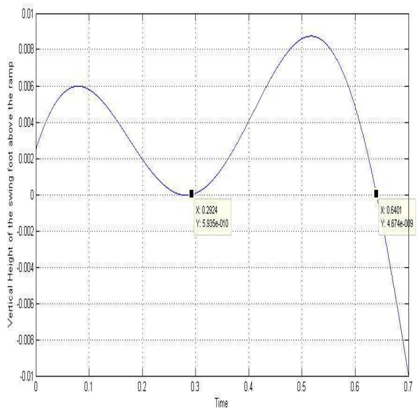

Figure 4: The State Space Position of Foot of a Swing Leg of PDBR in First Step

By examining figure 4, the vertical height h of a foot of a swing leg is zero at time t=0.2924 and a state-space position of robot at that instant is $y(t)$ = [0.0062; 0.0062; −0.2202; 0.8539], that means, both legs are in the same position so it does not consider as the impact. However, the actual impact time of heel strike is τ=0.6401 when the vertical height of a foot of swing leg is zero and new state-space position is $y_1(\tau)$ = [−0.1144; 0.1144; −0.6067; −0.6373]. Moreover, the orientations of leg-angles of PDBR for the first step are shown in figure 5.

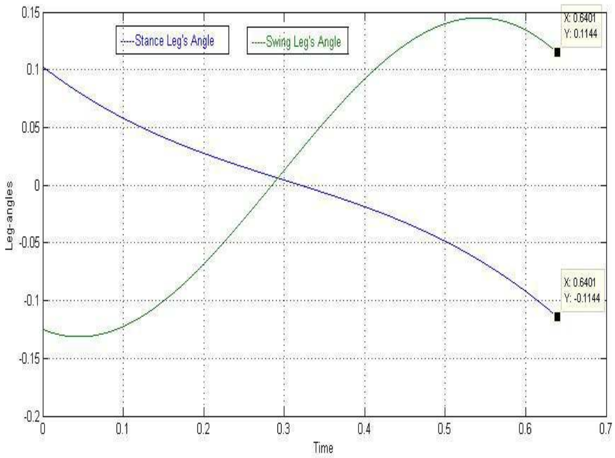

Figure 5: Orientations of Leg-Angles of PDBR in the First Step

Appling the transition equation T(φ) on a state-vector $y_1(\tau)$, then a new state vector

$$y_1^+(\tau) = T(\varphi)\, y_1(\tau).$$

will be the initial vector for the second step and repeating this procedure for second step. As the system grow, the foot displacements of a swing leg demonstrated in figure 6.

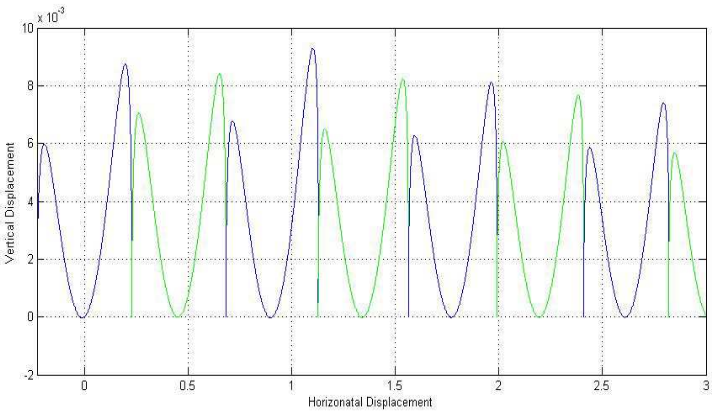

Figure 6: Displacements of Foot of Swing Leg of PDBR during the Walking on the Inclined Ramp

In addition, the figure 7 displayed the orientations of stance leg angle, swing leg angle and transition phase of PDBR.

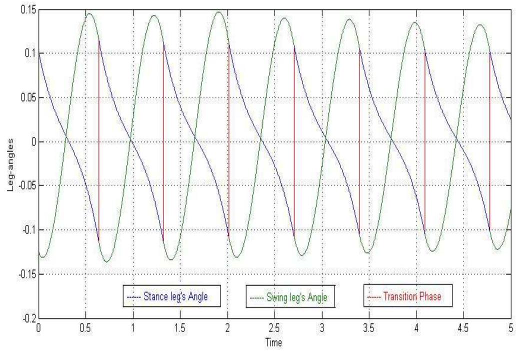

Figure 7: Blue Lines Show the Orientation of Stance Leg, Green Lines Show the Orientation of Swing Leg and Red Lines, Transition Phases at the Time of Impacts

Simulation results of this analysis are displayed in Table 3.

Table 3: Impact Time τ, Inter-Leg Angle φ, the Maximum Vertical Height of Foot of Swing Leg and Distance Travel by PDBR in Each Step

tep No	Impact Time	Inter-leg Angle(φ Rad)	Maximum Vertical Height of Swing Foot (10e-3m)	Distance Travel by PDBR(m)
1	0.6401	-0.2287	0.87418	0.4554
2	1.3217	-0.2261	0.84111	0.9092
3	2.0183	-0.2199	0.93034	1.3543
4	2.7067	-0.2161	0.8223	1.7901
5	3.3986	-0.2116	0.81332	2.217
6	4.0887	-0.2078	0.76696	2.6357
7	4.7794	-0.204	0.74119	3.0469
8	5.4698	-0.2005	0.71098	3.4495
9	6.1597	-0.1972	0.68148	3.8465
⋮	⋮	⋮	⋮	⋮
21	14.54659	-0.098617	0.207342	8.341356
22	15.26735	-0.077102	0.100357	8.721832
23	**-----**	**-0.052239**	**-0.025874**	**---**

Table-3 shows that the PDBR walks up to 22 steps for 15.26735 sec. During this walking, it travelled 8.721832 meter on the inclined ramp with the initial condition y_0. In the 23rd step, we observed that it falls down as the value of the inter leg angle less than 2α at the time of impact.

CONCLUSIONS AND DIRECTION FOR THE FUTURE

Our research shows the graphical approach to see the walking of the passive dynamic robot. It gives an idea about the stepwise analysis of the linearized model of the passive dynamic robot with respect to the initial condition. It demonstrates that how the graphs, the vertical height of a foot of the swing leg versus time, leg angles versus time, vertical height versus horizontal displacement of the foot of swing leg helps to understand the flow of the system. With assumed initial condition and parameters, our robot travelled 8.721832 meter. Our intension is to increase this distance by changing

the initial value as well as changing values of parameters and aim to analysis the stability of the passive dynamic robot.

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